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Entropy and Symmetry — Their Relation to Thought Processes in the Biological System

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Abstract. This paper addresses the topic of the electrophysiology of thought processes in a biological system. A classification and criticism of definitions of "intelligence" is presented. The engenderment of "intelligent" regulations of behavior is seen to involve stipulations concerning the relation of metron to logon informational content (MACKAY, 1950). The relationship is of a nature similar to that hidden behind the Maxwell demon (MAXWELL, 1871), and to certain difficulties already faced in quantum mechanics. As thought structures observe conservation of variety under transformations, the conservation of entropy in thermodynamics is investigated. Conservation occurs in a Carnot cycle (CARNOT, 1824) and only under a Lorenz transformation subject to a symmetry group formation. The symmetry group is topic neutral and could be applied to informational structures. The need for the informational equivalents of a Carnot cycle and Lorenz transformations is described. To depict the interaction of energy and structure, equivalents of EINSTEIN's field equations are needed. The study of symmetry groups by electrophysiological methods is seen to be at least feasible.

Introduction

In a general sense it may be said that if one considers the strategies of adaptation used by organisms, their derivation may be attributed to one of three processes (PIAGET, 1950, p. 12):

(1) The means of adaptation exist already within the environment and the organism has merely to adopt, ready-made, the processes already existing. A theorist of this kind is SPEARMAN (1927), who postulated the operation of intelligence to be the "apprehension of experience" and "the eduction of relations and correlates", i.e., a taking advantage of already existing relations. Another is RAVEN (1958), who holds the operation of intelligence to be the capacity to grasp the "necessary information and the capacity to form comparisons". It may be asked how, without prejudging the issue, "necessary information" can be grasped without intelligent deliberation of what is, indeed, "necessary". HEIM (1954), also, states that intelligence is the power of "grasping the essentials and responding appropriately" and, once again, one may question whether it is not more intelligent to decide what is "essential" than to grasp it, i.e., that which is "essential" must be relative to a course of action rather than autonomously existing. Those who believe that the brain functions like a present-day computer which requires programming also belong in this class. By and large, this derivation is empiricistic and Lamarckian.

(2) The means of adaptation exist already within the organism and there is a mere "trying out" of hypotheses upon the environment. Such a theorist is THORNDIKE (1927), who correlates intelligence with "the power of good response from the point of view of truth or fact". One may wonder, in the case of THORNDIKE's definition, whether a response is "good" prior to "truthful" and "factual" feedback, or only after; if before, then why bother with the feedback; if after, then the organism is blindly groping without foresight. KNIGHT (1933) equates intelligence with the

"capacity of relational constructive thinking directed to the attainment of some end". With this definition one may remark that the question is begged with the use of the adjective "constructive" and it would appear that a process is described in arrival terms. These derivations are pragmatic.

(3) The means of adaptation are a fabrication of organismic activity and environmental feedback. A theorist of this kind is PIAGET (1950), who states that "behavior becomes progressively more intelligent the more complex the lines of interaction between organism and environment". Another is VERNON (1960), who states: "We have arrived at the view that intelligence corresponds to the general level of complexity and flexibility of a person's schemata which have been built up cumulatively in the course of his life time. It would follow that no sharp distinction should be drawn between intelligence and attainment". We shall take our start from this class of definition: namely, that the nature of reality is a fabrication of organism and environment; that the biological system, if it is a computer, is a self-programming computer. From considerations of entropy and symmetry it will be shown that this class of definition is correct. A by-product of this aim will be to show exactly the logical connection between orectic and cognitive functioning.

Not included in the three categories above are those theorists who commence their definition with the words "ability" or "capacity", etc. Such definitions usually substitute one definition word for another and advocate a policy or view on intelligence. They must, therefore, be considered *ex cathedra*. TERMAN (1916), for instance, correlates intelligence with the ability to "carry on abstract thinking". WECHSLER (1958), also, states that intelligence is the "aggregate or global capacity of an individual to act purposefully, to think rationally and to deal effectively with his environment". GARRETT (1946) is of this kind, for whom intelligence is found in "abilities demanded in the solution of problems which require the comprehension and use of symbols". First, these definitions consider the nature of intelligence to be a resultant effect rather than a process (therefore they might be included in (2)), and second, there is a confusion of an invariable concomitant with the thing to be defined.

The contribution of HEBB (1949) has been to make a distinction between intelligence *A* (innate potential) and *B* (average level of performance), which appears to mirror the genotype-phenotype distinction. His case is supported from evidence in the case of brain injury. An I.Q. of 160 is possible with the loss of one frontal lobe (HEBB, 1939) and of 115 after hemidecortication (ROWE, 1937). On the other hand, early brain injury will have a severe effect on later mental growth (HEBB, 1942). His theory has prompted research with rats and chimpanzees indicating that the learning of the mature animal owes its efficiency to the slow and inefficient learning that has gone before, although limited and contained by it (RIESEN, 1947; FORGUS, 1954; RIESEN, 1958). If, as HEBB indicates, the organism's success at adapting is dependent upon a growth of structure of phenotypical form, then the arguments from empiricism and pragmatism do not hold water. For the "structure" was a joint enterprise of the organism and its environment.

A Bridge Between the Orectic and Cognitive Realms

If the previous analysis of intelligence is correct, then the engenderment of effective schemata entails meeting a certain criterion. This criterion of effectiveness must be of an analog or metrical nature. Thus, for example, the structural or cognitive aspects of behavior regulation could be considered to be purely digital, and we can examine the relation of analog processes to the formation of these digital aspects. The interdependency is of a nature reminiscent of MACKAY's (1950) distinction between the metron and logon content of information. Another analogy would be the interdependency of velocity and state in quantum mechanics, which is similar to the interdependency of the energic and the structural in the formation of brain structures. The analog properties of the brain within the confines of structural containment by thermodynamic principles seem to affect the formation of those structures themselves. If this is so, then there is the experimental problem of explaining how. Before this problem is attempted (and it will not be in this paper), we shall turn to the problem of explaining why. The reason why the analog properties of the brain — acting within the confines of structural containment by thermodynamic principles — affect the formation of those structures themselves, is that there exists a Maxwellian demon within the brain. Maybe more than one. MAXWELL's demon concerned himself with the interdependency of thermodynamic logon content and information theory logon content. The overall logon content or degrees of freedom of the system remains, of course, the same. This kind of equilibrium, between the thermodynamic and informational realms, is mirrored in the equilibrium between the logon and metron content in both realms. It would appear, therefore, that MAXWELL's demon requires two lieutenants on the logon-metron content borderlands. So the troll on the bridge between the orectic and the cognitive realms is MAXWELL's demon, who is bound by a set number of degrees of freedom and is entirely a logon-logon troll. His two lieutenants exist in both realms, each bound by their own set of degrees of freedom and they are logon-metron trolls. The lieutenant in the cognitive realm presides over the formation of cognitive schemata. It is interesting to speculate whether the three demon-trolls are themselves bound in equilibrium by a finite number of degrees of freedom. As a profound believer in the ultimate nature of the laws of thermodynamics, the writer presumes that they are.

The Lieutenant in Information Theory to Maxwell's Demon

In 1871 JAMES CLERK MAXWELL conceived ...

"... a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are still as essentially finite as our own, would be able to do what is at present impossible to us. For we have seen that the molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, *A* and *B*, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from *A* to *B*, and only the slower ones to pass from *B* to *A*. He will thus, without expenditure of work, raise the temperature of *B* and lower that of *A*, in contradiction to the second law of thermodynamics." (MAXWELL, 1871, pp. 328—329.)

It is agreed that SZILARD (1929) first solved the paradox by indicating the equilibrium condition of entropy (in thermodynamics) and negative entropy (in later-to-be-called information theory) within the constant sum of the overall system. The demon, therefore, merely withdrew "money" from one bank account to place it in another, the overall sum of money in the two accounts remaining the same with no spending and no deposits. Thus, it may be said that MAXWELL's demon increased the temperature in one portion of the vessel at the expense of information (negative entropy). We shall concern ourselves in this section with a similar being in the brain, who "cashes" his metron content for negative entropy. PAPERT's (1963) hypothetical machine, the génétron, illustrates this process.

It is a feature of the psychology of PIAGET that cognitive structures tend towards an equilibrium by an ontogenesis of successive equilibrations. PAPERT attempts to show how this equilibration process may be simulated. He begins his monograph with an exposition of the term "equilibrium", and his thesis is directed against BRUNER's (1959) review of INHELDER and PIAGET's book (1958). BRUNER's complaint is that:

"... if it be the case that the stage of concrete operations develops an equilibrium at an earlier stage, how comes it that the child gets beyond this stage to that of formal operations. Indeed, what is it that impels development in one direction rather than some other?" (p. 369.)

PAPERT points out that a process of equilibration finishes by creating the conditions for a new state of disequilibrium. It is as if the organism, having organized itself to take advantage of restraint in part of the environment, finds such a strategy unable to adapt to variety in a wider environment. The notion of a wider environment may be linked with that of "span of apprehension" or channel capacity of an organism.

Even although operational structures are needed, this is not a sufficient condition for their development. The point of difference is that BRUNER wishes to use "equilibrium" as descriptive of the final stage of thought without reference to process of engenderment, whereas PIAGET wishes to describe both the process (equilibration) of the development of thought and the end state (equilibrium).

PAPERT's definition of equilibrium is enlightening (p. 145): A system S of varying state has state $S(t)$ at time t. The perturbations $\Delta_{S(t)}$ are composed of a number of subprocesses, $\delta_1(t) \ldots \delta_n(t)$ summing to $\Delta_{S(t)}$. External influences $E(t)$ induce the perturbations $\delta_{S(t)}$ as a function of the state $S(t)$ and the influence $E(t)$:

$$\delta_i(t) = \delta_i(S(t), E(t)).$$

If the system at time t_1 arrives at an absolutely stable state, e.g., $\delta_i(t_1) = 0$, whatever $E(t)$, then the state $S(t_1)$ is a state of equilibrium.

Two quite different hypotheses about the evolution of the system are examined:

$$\begin{aligned} \delta_i(t) &= \delta_i(E(t)), \\ \delta_i(t_1) &= 0, \qquad t \neq t_1, \end{aligned} \tag{1}$$

$$\delta_i = \delta_i(S(t)). \tag{2}$$

(1) describes a situation in which the modifications of the system are a function of the external influence. (2) describes a situation in which the external influence plays no role in the evolution of the system.

Now, PAPERT wishes to apply the term "process of equilibration" to the second of these two cases rather than to the first. BRUNER and fellow empiricists are accused of relegating less autonomy to the "degree of emergence", i.e., in extreme cases, the empiricist argument entails $S(t_1)$ solely as a function of the diverse influences $E(t)$. PAPERT and PIAGET are Darwinian: a rejection of empiricism entails the acceptance of a form of emergentism. The emergence is, of course, a precipitate of organismic-environmental interaction. For PAPERT, it is a disproof of the theory that the evolution of logical structures is by a succession of additions to the repertoire available to the subject. It cannot be, therefore, that there is a progressive "eroding away" of the *tabula rasa* or even that there is a *tabula rasa*.

The description of PAPERT's model will be postponed for the time being in order to address a subargument: why does a two-year-old child not grasp logical operations even if suitably instructed? Why are there stages in thought? It possibly has something to do with the magical number seven (MILLER, 1956), the supposed "ultimate apprehension span", which is far from being seven at the age of two. The span of apprehension displays a growth from childhood to adolescence. MCLAUGHLAN (1963) has shown how this increasing propensity for conceptualising is directly correlated with memory span. The span appears to be of memory "chunks" to use MILLER's term or "foci of organization" to use LASHLEY's (1954, p. 430), available for simultaneous processing. The organism acts as a filter (BROADBENT, 1958) which expands from infancy to adolescence. Memory capacity and attention are considered closely related.

MCLAUGHLAN proceeds as follows: from the premiss that problem solution involves considering all other classes defined by the attributes of the given classes and if N equals the number of attributes of classes considered, then each relevant distinct class is defined by one of 2^N possible combinations of the presence or absence of those attributes. Using MCLAUGHLAN's analysis: at level 0, the demand is for the capacity to process $2^0 = 1$ concepts at a time (i.e., memory span is 1); at this level there can be no inferences drawn as attention is "filled" by one concept. At level 1 there is the capacity to retain $2^1 = 2$ concepts simultaneously — memory span is 2. Class concepts can now be formed but no distinction can be made between more than one object and its environment (which would entail three concepts). At level 2 there is the capacity to process up to $2^2 = 4$ concepts — with a memory span of 3—4 digits. Finally, at level 3 there is the ability to process up to $2^3 = 8$ concepts simultaneously with a memory span of 5—8 digits. This final stage corresponds to the formal operational intelligence (11—15 years) of PIAGET.

To simulate a process without building into the model preformistic tendencies is difficult; simulation of the end result is not as difficult as a correct ontogenetic simulation. PAPERT's model, to be described below, fails in that other measures are used for the step-by-step elaboration of cognitive operations than the progressively increasing capacity to process informational chunks simultaneously. The narrow capacity of the infant is not simulated by a virgin génétron; time plus the relevant experience are all that is needed for the génétron to arrive at formal operational intelligence, but no amount of time or coaching will equip an infant of a memory span of under 3 digits with formal operations — it does not have the channel capacity.

PAPERT considers three stages in the growth of logical thought: (1) pre-operational thought; (2) concrete operations; and (3) formal operations. The judgment of the infant at stage 1 is based upon a perceptual index which, for the infant, is prepotent over other indices. This prepotency is determined by a process of probabilistic selectioning. The Piagetian term "index" (PIAGET, 1951) is used rather than "stimulus" and would appear to be similar to DEWEY's (1896) definition of a stimulus arising out of a sensori-motor co-ordination:

"... what precedes the "stimulus" is a whole act, a sensori-motor co-ordination. What is more to the point, the "stimulus" emerges out of this co-ordination; it is born from it as its matrix; it represents as it were an escape from it."
"The stimulus is that phase of the forming co-ordination which represents the conditions which have to be met in bringing it to a successful issue; the response is that phase of one and the same forming co-ordination which gives the key to meeting these conditions, which serves as instrument in effecting the successful co-ordination."

This viewpoint is reflected in the writings of later day authors (cf. MILLER, GALANTER and PRIBRAM, 1960).

An example is chosen which requires the ability to hold two concepts simultaneously in attention. Suppose three index instigated sequences: $I_1 \rightarrow R_1$; $I_2 \rightarrow R_2$; and $I_3 \rightarrow R_3$. Stage 2 occurs when the reactions $I_1 \rightarrow R_1$ and $I_2 \rightarrow R_2$ are combined as an element of a group to give an outcome of compensation when serially applied. Formally the successive application of the two transformations may result in a zero transformation of the operand. The realization is that transformation involved in the experiment are *one-one* (cf. ASHBY, 1957, p. 135) and that *variety is conserved by a one-one transformation*:

"... if messages of variety v are to pass through several codes in succession, and are to be uniquely restorable to their original forms, then the process must be one that preserves the variety in the act at every stage." (ASHBY, 1957, p. 142.)

The infant, of course, must be able to compare the variety after one transformation, with the variety after another (i.e., to hold two concepts simultaneously in attention). Variety is, of course, related to the notion of degrees of freedom.

PAPERT's génétron combines the plasticity of ROSENBLATT's (1958) perceptron with the equilibrating properties of a homeostat. The output of perceptrons of a lower order is fed into those of a higher order resulting in transformation rules of a higher order. This is illustrated in the more simple learning of compensatory strategies and negative feedback, the model proceeding by stages of compensation and not by prior programming. There is no growth of cells in the machine (corresponding to an increasing span of apprehension), so the model would appear to simulate an adult, reared in seclusion, confronted with a logical problem for the first time. Connections are pre-established supposing an organism that receives the relevant information in the correct order. Memory for the perception is a "preference for a particular response", which, although insufficient for explaining perceptual recognition, is sufficient for thought simulation. Modification without external feedback is accomplished by equipping the machine with two types of cells A and F; cells F are those of the hierarchically arranged perceptrons; cells A modify their action if the inputs $x_1 \ldots x_n$ satisfy often enough the function $F(y_1 \ldots y_n) = 1$, (y_i being the output to other cells); in this way logical "concepts" are formed. The cells F are sensitive to disequilibrium in the machine and through feedback connections within the machine reduce the complexity of information. The machine acts so that (p. 151) for the function $R^* = h(R_1' \ldots R_n')$ to develop with effects different from $R_1' \ldots R_n'$, it is sufficient to have an approximation to every function R_i' to permit the evolution of R^*. If the system is disposed already to an ensemble of N first stage functions, the probability of the machine arriving at R^* defined in terms of the first stage of the hierarchy, because of overlapping connections, augments rapidly with N.

Now, it should be noted that the cells F of PAPERT's génétron are sensitive to a metron informational content, and that cells A modify when $F(y_1 \ldots y_n) = 1$. Cells A, therefore, are "engenderers of logon content".

In information theory, then, one finds a demon who performs a function analogous to that performed by MAXWELL's demon in the more general realm of thermodynamics. In the engenderment of information structures, regulation by metron content is replaced by that of logon content and this lieutenant-demon must obey the conservation of variety in its replacements.

The Nature of Reality — The Lair of Devils

I will attempt to show in this section that the meet of entropy (thermodynamic) and negative entropy (information) is the same as the meet for two interacting physical quantities. The physicist, it will be seen, pays a price for his predictions; similarly, devils pay a price for their use of information.

In 1935 EINSTEIN, PODOLSKY and ROSEN asked if a quantum mechanical description of physical reality could be considered complete. Since, in the case of two physical quantities described by non-commuting operators, knowledge of one precludes knowledge of the other (implying that these two quantities cannot have simultaneous reality), quantum mechanics cannot be complete. When the momentum of a particle is known, its coordinate is not predictable and may be obtained only by a direct measure. With such a measure, the particle is disturbed and its state is altered. If the operators corresponding to two physical quantities do not commute, then the precise knowledge of one of them precludes such a knowledge of the other.

A similar thought might occur to an experimental epistemologist. How is it possible to know the "real" world, if the means whereby the input from this world is processed arose (as I showed in the previous section) by interaction with this very world? Let us suppose that the two "physical quantities" described below are an epistemologist and his environment, or the two portions of MAXWELL's vessel.

An immediate reply to EINSTEIN, PODOLSKY and ROSEN came from BOHR (1935) who objected to their criterion of reality:

"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." (Einstein, Podolsky and Rosen, 1935, p. 777.)

Bohr's point is:

"... the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails — because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose — the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality." (p. 697.)

Thus the meaning of Einstein et al. "without in any way disturbing a system" can be termed ambiguous. For one must take into account the influence exerted by the measurement on the very conditions which determine the possible types of measurements. Bohr's viewpoint of "complementarity" of physical quantities obviates the need to consider quantum mechanics as providing an inadequate description of physical reality:

"... it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena, that the notion of *complementarity* aims at characterising." (p. 700.)

The unambiguous use in quantum theory of the concepts of position and momentum implies their mutually exclusive character. Bohr considers that the discrimination in every experimental arrangement between those parts of the physical system considered measuring instruments and those the objects under investigation may be said to form the principle distinction between classical and quantum mechanical descriptions of physical phenomena. This being the case, one might hold that the relation of entropy (thermodynamic) to negative entropy (information) is quantum mechanical, as is that between the epistemologist and his environment.

Schrödinger (1935) called this *the* characteristic trait of quantum mechanics:

"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own." (p. 555.)

The procedure whereby one system is known separately from the other is known as the "disentanglement". Schrödinger points out that the process involves a regressus ad infinitum, since the procedure itself involves measurement.

Schrödinger proves that the representative arrived at for one system depends on the *programme* of observations to be taken with the other one. x and y stand for all the coordinates of the first and second systems respectively and $\Psi(x, y)$ for the normalized representative of the state of the composed system, when the two have separated again, after the interaction has taken place. After the performance on the second system of certain observations so that its representative at disentanglement is sure to turn up as one out of the known complete set of normalized orthogonal functions $f_n(y)$, if all the variables measured commute, $\Psi(x, y)$ may be developed into a series with respect to the f_n,

$$\Psi(x, y) = \sum_n c_n g_n(x) f_n(y) \tag{1}$$

so that the representative of the other system may be known. If measurements on the y-system indicate $f_k(y)$, then $g_k(x)$ is the representative of the x-system. The c_k enter in order that the g_k may be assumed normalized:

$$\int g_k^*(x)\, g_k(x)\, dx = 1 . \tag{2}$$

The equation:

$$c_k g_k(x) = \int f_k^*(y)\, \Psi(x, y)\, dy \tag{3}$$

together with (2) determine the c's and g's.

Schrödinger admits no reason for the g_k to be orthogonal to each other, but asks *when* they are, i.e., *how* must the f_k be chosen for that purpose? The condition for this is:

$$c_k^* c_l \delta_{kl} = \int dx \int dy \int dy'\, f_k(y')\, \Psi^*(x, y')\, f_l^*(y)\, (x, y) . \tag{4}$$

Or, for every k, the function:

$$u_k(y) = \int dx \int dy'\, f_k(y')\, \Psi^*(x, y')\, \Psi(x, y) \tag{5}$$

is to be orthogonal to all the $f_l(y)$, with the possible exception of $f_k(y)$. Thus $u_k(y)$ must be a numerical multiple of $f_k(y)$. From (4) with $l = k$, it is seen that the numerical multiplier is $|c_k|^2$. Therefore:

$$|c_k|^2 f_k(y) = \int dx \int dy'\, f_k(y')\, \Psi^*(x, y')\, \Psi(x, y) . \tag{6}$$

Introducing the function:

$$K(y, y') = \int dx\, \Psi^*(x, y')\, \Psi(x, y) \tag{7}$$

which has Hermitian symmetry, it is seen from (6) that the reciprocals of the $|c_k|^2$ and the functions $f_k(y)$ are required to be the eigenvalues and a system of eigenfunctions respectively of the homogeneous equation

$$f(y) = \lambda \int K(y, y')\, f(y')\, dy' . \tag{8}$$

Provided that the integral in (7) converges, so that K is defined, a complete solution of (8) exists. By using the eigenfunctions for the development of (1) one sees that the λ_k^{-1} *are* all non-negative and sum to unity.

The general case is that all the λ_k^{-1} are different from one another except maybe for an arbitrary set of them vanishing. Then the *relevant* $f_k(y)$ are uniquely determined and so are the $g_k(x)$. Hence there is always *one* development of $\Psi(x, y)$ of the type which might suitably be called "biorthogonal".

The $f_k(y)$ and $g_k(x)$ have a mutual implication, as is revealed in Schrödinger's proof of non-invariance in the response of the other system:

"Whenever (and of course only when) the eigenfunctions of a programme to be carried out on the y-system include the relevant functions $f_k(y)$, or the eigenfunctions properly speaking of (8), the programme will lead to the biorthogonal development and imply the relevant $g_k(x)$ as the other set. Now if for an arbitrarily fixed programme of measurements on the y-system the representative arrived at for the x-system was the same *in all individual cases*, the same $g_k(x)$ would have to turn up (and even with the same probabilities) as in the biorthogonal development; for in two infinite series of repetitions *ab ovo* of one and of the other programme respectively every possible result occurs according to its due probability. Hence the relevant functions $g_k(x)$ would have to be implied whatever programme is carried out. But since, of course, they also determine the biorthogonal development uniquely and thereby require the relevant $f_k(y)$ as the other set, these would have to be included in the eigenfunctions of every programme which cannot be, since the latter are, by principle, an entirely arbitrary complete orthogonal set. Hence the non-invariance is proved." (pp. 557—558.)

Thus the entanglement consists in that one and only one observable (or set of commuting observables) of one system is uniquely determined by a definite observable (or set of commuting observables) of the other system. Thus there is always one and as a rule only one development of the normalized representative of the state of the composed system, when the two have separated again and after the interaction has taken place.

If one considers, therefore, the two systems in question to be either:

(1) a system of negative entropy (information) and a system of entropy (thermodynamic), or

(2) the external environment and the epistemologist, then one might say that the entanglements are not arbitrary. It is also true that our knowledge of the external world is of the nature of $\Psi(x, y)$ and if we are each an "x", then we can never know a "y" — which could be a reason for believing in the noumenal world of KANT! Again, the nature of the amalgams of cognitions and "wants" known generally as "intentions" must also be of the nature of $\Psi(x, y)$. So, if our "wants" are "x's", then the various means we have for satisfying these "wants" which are stored in our memories, are "y's". So far, all the computers built have been "y"-machines. Only when a $\Psi(x, y)$ machine is built will the biological organism even begin to be approximately simulated. For it is of the nature of the organism to be $\Psi(x, y)$.

Relativity in Thermodynamics and Information Theory

In this section I will attempt to show that relativity must be taken into account in describing thought processes. We shall begin by introducing the terminology of TOLMAN (1934) where:

E = energy; S = entropy; Q = heat; W = work; v = volume; p = pressure; F = thermodynamic potential; H = heat content; A = free energy; T = temperature; $\boldsymbol{E}$ = electric field strength; and $\boldsymbol{H}$ = magnetic field strength.

Then: $\Delta E = Q - W$ for the energy change of a system in terms of heat absorbed and work done, and $\Delta S \geqq \int d\frac{Q}{T}$ for the entropy change in terms of heat absorbed and temperature. Considering the Lorenz transformation for physical quantities, the transformation equation for entropy is simply:

$$S = S_0 .$$

This is because if a thermodynamic system at rest with the entropy S_0 is accelerated to the velocity u reversibly and adiabatically without change in its internal state, then there is no change with respect to the coordinate system in its entropy S. This agrees with the interpretation of statistical mechanics of entropy in terms of probability, since the probability of finding a system in a given state should be independent of the velocity of the observer relative to it.

Cognitive structures tend towards an equilibrium of compensatory operations. For this reason it is possible to talk of "equivalent observers" within the electrical activity of the mammalian brain. Animals tend to take short cuts to their destination. By the fact that "equivalent observers" are possible, then the following two principles of physical phenomena must find application in the biological study of nervous systems: (1) *The Principle of Covariance*: physical laws can be expressed in a form which is independent of the coordinate system. (2) *The Principle of Equivalence*: there is a corrrespondence between the result which would be obtained by an observer who makes measurements in a gravitational field using a frame of reference which is held stationary, and the results obtained by a second observer who makes measurements in the absence of a gravitational field by using an accelerated frame of reference.

To make my point clear at the beginning: I am suggesting that what gravity is to the cosmologist, knowledge or information context is to the experimental epistemologist; both provide context. What velocity is for the cosmologist, value or temperature or the degree to which essential variables in the system have exceeded their limits, is for the student of thermodynamics. In defence of the former equation: there seems no reason why one cannot, at a level of abstraction, equate a physical system with its gravitational context. Similarly, a biological regulative structure of the nervous system ("idea") may be equated with informational context. In the defence of the latter equation: I see no reason why one cannot equate the phenomenological term "value" with the physical description of how much it takes to bring a system to equilibrium. If this is correct, then one has a measurement which is relative to the system considered and this is the case with velocity and also with temperature.

The "*Mach Hypothesis*" proposes the idea that the geometry of space-time is determined by the distribution of matter and energy, so that some kind of field equations connecting the components of the metrical tensor $g_{\mu\nu}$ with those of the energy momentum tensor $T_{\mu\nu}$ are implied.

Now the Riemann-Christoffel tensor expressed by the equation:

$$R^{\tau}_{\mu\nu\sigma} = \{\mu\sigma, \alpha\}\{\alpha\nu, \tau\} - \{\mu\nu, \alpha\}\{\alpha\sigma, \tau\} + \frac{\partial}{\partial x^{\nu}}\{\mu\sigma, \tau\} - \frac{\partial}{\partial x^{\sigma}}\{\mu\nu, \tau\}$$

where: μ = magnetic permeability; σ = electrical conductivity; α = degree of dissociation; ν = frequency; τ = period; is constructed solely from the components of the metrical tensor $g_{\mu\nu}$. The condition:

$$R^{\tau}_{\mu\nu\sigma} = 0$$

is sufficient and necessary condition for the validity of the special theory of relativity, as the components of $g_{\mu\nu}$ are then constants and gravitational fields are transformed away by a suitable choice of coordinates. To obtain a less stringent condition for the case of a gravitational field in the *empty* space in the neighborhood of gravitating bodies the contracted Riemann-Christoffel tensor is obtained by setting $\sigma = \tau$ and summing:

$$R_{\mu\nu} = \{\mu\sigma, \alpha\}\{\alpha\nu, \sigma\} - \{\mu\nu, \alpha\}\{\alpha\sigma, \sigma\} + \frac{\partial}{\partial x^{\nu}}\{\mu\sigma, \sigma\} - \frac{\partial}{\partial x^{\sigma}}\{\mu, \sigma\}.$$

The relativistic analog of Poisson's equation:

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}+\frac{\partial^2 \Psi}{\partial z^2}=4\pi k\varrho$$

which in the Newtonian theory of gravitation connects the single gravitational potential Ψ with the density of matter ϱ and the gravitational constant k was proposed by Einstein to be:

$$R_{\mu\nu}-\tfrac{1}{2}R g_{\mu\nu}+\Lambda g_{\mu\nu}=-kT_{\mu\nu}$$

where $R_{\mu\nu}$ is the contracted Riemann-Christoffel tensor, R is the invariant obtained by the further contraction of the tensor, Λ is the cosmological constant, k is a constant which is related to the ordinary constant of gravitation.

This equation connects the ten gravitational potentials of $g_{\mu\nu}$ and their derivatives with the components of the energy momentum tensor $T_{\mu\nu}$. It satisfies the principle of covariance by being a tensor equation valid in all systems of coordinates if valid in one.

From the electromagnetic stresses p_{ij} and the densities of electromagnetic mass ϱ and of momentum g_i, the electromagnetic energy-momentum tensor $[T^{\mu\nu}]_{em}$ can be constructed (Tolman, 1934, p. 99):

$$[T^{\mu\nu}]_{em}\quad \begin{matrix} p_{xx} & p_{xy} & p_{xz} & [\boldsymbol{E}\times\boldsymbol{H}]_x, \\ p_{yx} & p_{yy} & p_{yz} & [\boldsymbol{E}\times\boldsymbol{H}]_y, \\ p_{zx} & p_{zy} & p_{zz} & [\boldsymbol{E}\times\boldsymbol{H}]_z, \\ [\boldsymbol{E}\times\boldsymbol{H}]_x & [\boldsymbol{E}\times\boldsymbol{H}]_y & [\boldsymbol{E}\times\boldsymbol{H}]_z & \frac{E^2+H^2}{2} \end{matrix}$$

where the Maxwell stresses have the values:

$$p_{ii}=-\tfrac{1}{2}(\boldsymbol{E}_i^2-\boldsymbol{E}_j^2-\boldsymbol{E}_k^2+\boldsymbol{H}_i^2-\boldsymbol{H}_j^2-\boldsymbol{H}_k^2),$$
$$p_{ij}=-(\boldsymbol{E}_i\boldsymbol{E}_j+\boldsymbol{H}_i\boldsymbol{H}_j).$$

These expressions are, of course, in the four-dimensions of space-time.

Tolman (1934, p. 119) has written:

"In connexion with the phenomenological character of thermodynamics it is also of interest to emphasize once more the phenomenological character of relativistic considerations. Indeed, the formulation of the first postulate of relativity, as a generalization of failures to detect the motion of the earth through a suppositious ether, has an interesting parallelism with the formulation of the second law of thermodynamics as a generalization of failures to construct perpetual motion machines of the second kind. And the formulation of the second postulate of relativity as expressing the results of measurements on the velocity of light from moving sources, has something in common with the formulation of the first law of thermodynamics as expressing the results obtained in measurements such as those on the mechanical equivalent of heat."

Similarly, if one can provide electromagnetic data to determine $[T^{\mu\nu}]_{em}$, then by an equation similar to Einstein's equation above, one might determine the electromagnetic equivalent of tensors similar to $g_{\mu\nu}$. This, I postulate, would be a physical description of cognitive structures related to the degrees of freedom of a logical structure. Thus, instead of reducing the electromagnetic field to geometry, the geometry of logical structures would be reduced to electromagnetism — which, incidentally, is the exact opposite to the aim of the unified field theory. It is helpful to remind the reader that $g_{\mu\nu}$ is a symmetrical tensor, i.e., $g_{\mu\nu}=g_{\nu\mu}$. The symmetry of logical structures thus would find a physical counterpart.

Now, heat content, free energy and thermodynamic potential may be defined according to the following relations:

$$H=E+pv,$$
$$A=E-TS,$$
$$F=E-TS+pv=H-TS.$$

If a system is kept at *constant temperature* and *constant pressure* then the following relation is obtained:

$$\Delta F=\Delta E-T\Delta S+p\Delta v$$

and the condition for thermodynamic equilibrium is when:

$$\delta F=0$$

i.e., when there is no thermodynamic potential. With $\Delta v=0$, $\Delta A=F$. It can be proposed that the condition for the occurrence of a symmetric tensor $g_{\mu\nu}$, is when there is no free energy and

$$\Delta E=k\Delta S.$$

If negative entropy (NS) is correlated with entropy, then

$$\Delta E=k\Delta NS$$

in these conditions. It then remains to investigate the conditions and for this one must turn to the Carnot Cycle.

Carnot's Cycle and the Conservation of Variety

Using the terminology of Guggenheim (1967) one may define a Carnot cycle (cf. Carnot, 1824) by the following steps: after a system is taken through a complete cycle of states its entropy remains the same at the end as at the beginning:

1. $\Delta S=0$.

At all stages the system is in equilibrium, so no irreversible changes take place and:

2. $\Delta S=\sum_i q_i/T_i$ where q_i is the heat absorbed by the system.

Substituting 2. into 1.:

3. $\sum_i q_i/T_i=0$ which may be replaced by:

4. $\sum_r q_r/T_r=\sum_s Q_s/T_s$

where each q_r is a positive quantity of heat taken in at the temperature T_r and each Q_s is a positive quantity of heat given out at temperature T_s.

The work done is:

5. $-w=\sum_i q_i=\sum_r q_r-\sum_s Q_s$

and the ratio η defined by:

6. $\eta=-w\Big/\sum_r q_r=\left(\sum_r q_r-\sum_s Q_s\right)\Big/\sum_r q_r$
$\quad=1-\sum_s Q_s\Big/\sum_r q_r$

is called the *thermodynamic efficiency* of the cycle.

Suppose that there is a maximum temperature $T_{\max}$ and a minimum temperature $T_{\min}$ between which the cycle is confined. Subject to this restriction on the temperatures, the maximum possible value for η is obtained by making:

7. $T_r=T_{\max}$ (all r),

8. $T_s=T_{\min}$ (all s),

The cycle thus consists of isothermal absorption of heat at T_{max}, isothermal emission of heat at T_{min}, and adiabatic changes from T_{max} to T_{min} and from T_{min} to T_{max}. This cycle would be a *Carnot cycle*.

Substituting 7. and 8. into 4. one obtains:

9. $\sum_r q_r/T_{max} = \sum_s Q_s/T_{min}$ (Carnot's cycle).

Substituting 9. into 6. one has:

10. $\eta = 1 - T_{min}/T_{max}$ (Carnot's cycle).

So far, of course, we have been dealing with thermodynamic expressions. To indicate the analogy in information theory, which I should like to draw, the following expression by Shannon (1948) is a definition for *redundancy* which should be compared with expression 10. for the Carnot cycle:

11. $R = 1 - \log_2 n/\log_2 N_{max}$ where the term $\log_2 n/\log_2 N_{max}$

is the definition for the term *constraint* (C).

Paralleling the conservation of entropy under transformations of reversible changes, one may point to the conservation of degrees of freedom under phase contrast and amplitude contrast changes (Gabor, 1961). This is also an information theory analysis.

My point is this: although one may draw the parallels between thermodynamics and information theory terms, if the fabrication of information structures proceeds in a way similar to that outlined by Papert, then equations similar to Einstein's field equations, which unite metrical tensors with energy momentum tensors, must be considered in order to find out how the thermodynamic may interact with the structural. It may be that the achievement of a Carnot cycle entails the fabrication of an adiabatic wall to conserve the cycle in the biological organism. Such a structure would consist of an unknown biological material. However, its electrophysiological observation could proceed in the following way:

It has been shown (Gabor, 1961) that an electrical structure described in terms of amplitude, frequency and phase, will retain the number of degrees of freedom available to it under transformation, provided that amplitude and phase operators are both available to provide compensatory operations. There is thus a conservation of degrees of freedom. The study, e.g., of cortical structures, with respect to frequency and phase could be achieved electrophysiologically. This would be a preliminary suggestion.

It has been pointed out (Darrow, 1942) that there is no conservation of entropy except under the restriction that all of the proceeds of the system and in the outside world are reversible. Further, no transfer of heat is reversible unless the body whence it comes and the body whence it goes are of identical temperature. One might recall, in this regard, that a system transmitting without information loss implies that both receiver and transmitter have an equal repertoire of signals. Giles (1964, Chapter 15) has proven that the assumption of a set of equivalent observers implies the existence of an abstract symmetry group, and an observer can determine the nature of this symmetry group by a method which depends only on the possibility of communication between observers. The thermodynamic implications of symmetry are that they can all be expressed in terms of a certain representation of the symmetry group. This leads us to the next section, where we shall examine the nature of symmetry and the consequences of the analogy between temperature and signal repertoire sets.

Temperature in Thermodynamics Considered Analogous to A Priori and A Posteriori Information

Only under a Lorenz transformation does there exist an entropy function which is invariant. Giles (1964) considered the thermodynamic implications of Newtonian space-time. By recognizing that symmetry amounts to the equivalence of all inertial reference frames by the principle of equivalence, the existence of moving systems must be taken into consideration. This entails components of content such as the three components of momentum. Giles shows that the equilibrium surface is determined as a function of two parameters only: the *non-translational energy* U and the *spin* σ. It follows that the eleven components of potential of an equilibrium state are functions of two parameters: a *temperature* T and an *angular velocity* ω.

The components of content (conserved quantities) are derived as follows: A is a thermodynamic system; p denotes the momentum of A and m its angular momentum about the origin with respect to a definite initial reference frame. Thus, the energy E and the components p_1, p_2, p_3 and m_1, m_2, m_3 of p and m are components of content. Let the vector r be the position vector of A's centroid. Multiplying r by the mass M an additive function of state is obtained which is not, however, constant for an isolated system. A linear combination w is formed of the two additive functions of state Mr and p thus: $w = Mr - pt$ where t denotes time. Since $p = M\,dr/dt$, w is not only an additive function of state but also a constant of motion, so that its components w_1, w_2, w_3 are components of content, and w is called the *moment of mass*. There are thus 11 linearly independent components of content: M, E, and the components of p, m, and w.

Let $S = S^+(E, p, m, w)$ be the equation of the equilibrium surface of a system A. Remembering that we are dealing with Galilean transformations, Giles shows that as the property of being in an equilibrium state is objective, i.e., independent of the observer, then the equation for S must continue to hold whenever the quantities S, E, p, m, w are transformed in accordance with a change of reference frame. But S is invariant, so the *value* of the function $S^+(E, p, m, w)$ must be invariant under such a transformation.

It is shown that this condition severely limits the form of the function S^+: let A_1 be an equilibrium state of content (E, p, m, w). With a new observer A_1 is at rest at the origin. By Newtonian dynamics of a system of particles, relative to this observer the content of A_1 is $(E - p^2/2M, 0, m - w \times p/M, 0)$. Thus $S^+(E, p, m, w) = S^+(U, 0, \boldsymbol{\sigma}, 0)$ where $U = E - p^2/2M$ is the *non-translational energy* of the system and $\boldsymbol{\sigma} = m - \overline{x} \times p$ is its *spin angular momentum*, i.e., the angular momentum about the centroid. The invariance of S^+ under rotations shows that S^+ can depend only on the magnitude of σ of $\boldsymbol{\sigma}$. Thus Giles shows that S^+ is a function of two parameters only: $S^+(E, p, m, w) = S_0^+(U, \sigma)$. σ is called the *spin* of the system.

Let

$$\frac{\partial S_0^1}{\partial U} = \frac{1}{T}, \quad \frac{\partial S_0}{\partial \sigma} = -\frac{\omega}{T}$$

so that, for any infinitesimal quasi-static state of A,

$$T\,dS = dU - \omega\, d\sigma$$

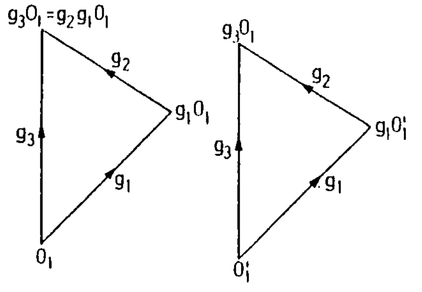

Fig. 1. By permission

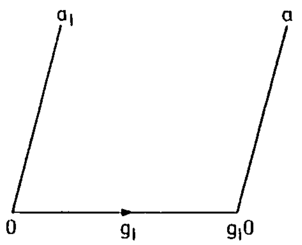

Fig. 2. By permission

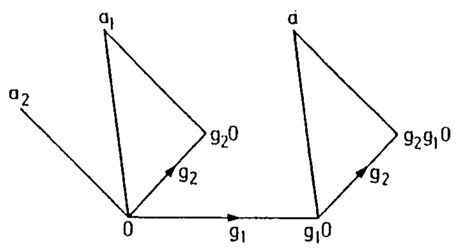

Fig. 3. By permission

where T is a temperature and ω an angular velocity. Then GILES indicates that for any equilibrium state there is a corresponding temperature T and an angular velocity ω which are quantities independent of the observer since they are functions of the invariants U and σ.

If the quantities ω and T suffice to determine the components of potential corresponding to $E, p_1, p_2, p_3, m_1, m_2, m_3, w_1, w_2, w_3$. These are denoted by $\varphi_E, \ldots, \varphi_{w}$, respectively and $\boldsymbol{\varphi}_p$ denotes the vector with components $\varphi_{p1}, \varphi_{p2}, \varphi_{p3}$ respectively and similarly for $\boldsymbol{\varphi}_m$ and $\boldsymbol{\varphi}_w$. Setting $\boldsymbol{\omega} = \omega\boldsymbol{\sigma}/\sigma$ and $\nu = p/M$, from the above equation for $T\,dS$ one obtains:

$$T\,dS = dE - (\nu - \boldsymbol{\omega} \times \bar{x}) \cdot dp - \boldsymbol{\omega} \cdot dm - \boldsymbol{\omega} \times \nu \cdot dw,$$

so that

$$\varphi_E = -1/T,$$
$$\boldsymbol{\varphi}_p = (\nu - \boldsymbol{\omega} \times \bar{\chi})/T,$$
$$\boldsymbol{\varphi}_m = \boldsymbol{\omega}/T,$$
$$\boldsymbol{\varphi}_w = \boldsymbol{\omega} \times \nu/T.$$

Let us return to our analogy. It was suggested above that temperature in thermodynamics corresponded to degrees of freedom in information theory. The angular velocity of the thermodynamic system may be equated with the electrical potential energy of the system. It remains to investigate the conceptions of symmetry in thermodynamics.

A set $\mathcal{O}$ of equivalent observers is postulated, it being assumed that these observers can communicate with each other. It is then shown how any observer can discover an abstract group $\mathcal{G}$ which characterizes the symmetry of the set $\mathcal{O}$ without regard to thermodynamics. GILES (1964, p. 151) states:

"Now, as WEYL (1952) has emphasised, whenever we are concerned with a symmetric structure it behoves us to investigate the corresponding transformation group. What is transformed by is not at first important; it is the abstract group itself which, in large measure, characterises the structure. Thus the symmetry of a sphere is described by the 3-dimensional rotation group, of Newtonian space-time by the Galilean group, of Minkowski space-time by the inhomogeneous Lorentz group, and so on. It seems desirable, then, that our primitive observer should at least be able to discover the abstract group which characterises the symmetry of his world."

GILES proceeds in a fashion which is more intuitive than mathematical. A set $\mathcal{O}$ of observers is postulated who can communicate with each other, also a "principle of equivalence" is decreed, whereby no observer occupies a distinguished position in the set $\mathcal{O}$. The assumption of free communication between observers implies that they should be able to distinguish between the other observers — in terms of messages received.

The problem of how an observer 0 can define and determine the structure of the symmetry group $\mathcal{G}$ which characterizes his world is next attacked. Let 0′ be any other observer and let g denote the appearance presented by 0′ to 0. Let $\mathcal{G}$ denote the set of all such g where 0′ ranges over the set $\mathcal{O}$ of all possible observers. The appearance which is presented to 0 by 0 itself is denoted by 1; this is the identity sign.

Suppose an observer 0_1, then there is one and only one observer whose appearance to 0_1 is g and this observer is denoted by $g\,0_1$. With every element g of $\mathcal{G}$ there is thus associated a transformation $0_1 \to g\,0_1$ of the set $\mathcal{O}$. These transformations form a group. In Fig. 1 (from GILES, 1964, p. 156), g_1 and g_2 are two elements of $\mathcal{G}$ and 0_1 any observer. g_3 is the appearance presented to 0_1 by $g_2\, g_1\, 0_1$, i.e., $g_3\, 0_1 = g_2\, g_3\, 0_1$. It should be noted that g_3 is independent of 0_1, rather $g_3 = g_2\, g_1$. The associative law is satisfied for $\mathcal{G}$, i.e., $(g_1\, g_2)\, g_3 = g_1 (g_2\, g_3)$.

A set $\mathcal{S}$ of states (denoted by $a, b, c, \ldots$) is postulated and the group $\mathcal{G}$ is realized as a group of automatisms of the set $\mathcal{S}$. A state a will present different appearances to distinct observers 0_1 and 0_2. Let a be any state and g_1 an element of $\mathcal{G}$; let a_1 be that state which presents to 0 the same appearance as does a to $g_1\, 0$ (cf. Fig. 2, from GILES, 1964, p. 158). The principle of equivalence implies that the correspondence $a \to a_1$ is a 1:1 mapping of $\mathcal{G}$ onto itself. This is denoted by $\theta(g_1)$, i.e., $a_1 = \theta(g_1)\, a$.

It is next shown that the mapping $g \to \theta(g)$ is a realization of the group $\mathcal{G}$ in terms of automatisms of $\mathcal{S}$, i.e., if g_2 is any other element of $\mathcal{G}$,

$$\theta(g_2)\, \theta(g_1) = \theta(g_2\, g_1).$$

This is accomplished by noting that $\theta(g_2)\, \theta(g_1)\, a = a_2$ must be determined. According to Fig. 3 (from GILES, 1964, p. 158). $a_2 = \theta(g_2)\, a_1$ is that state which presents to 0 the same appearance as a_1 does to $g_2 0$. But a_1 and $g_2 0$ present, respectively, the same appearances to 0 as a and $g_2 g_1 0$ do to $g_1 0$. By the principle of equivalence, the appearance presented by a_1 to $g_2 0$ is the same as that presented by a to $g_2 g_1 0$. Also, the observer $g_2 g_1 0$ presents to 0 the appearance $g_2 g_1$. Therefore $a_2 = \Theta(g_2 g_1) a$. As the state a was arbitrary, this means that $\Theta(g_2)\,\Theta(g_1) = \Theta(g_2 g_1)$ which completes the proof.

Functions of states may also be transformed. Let X be a proper additive function of state and g any element of $\mathscr{G}$. X_1 denotes that function of state which bears the same relation to 0 as X does to $g\,0$, i.e., X_1 is defined by

$$X_1 \cdot \Theta(g)\, a = X \cdot a$$

X_1 thus defined is a proper additive function of state and is a linear function of X. To every element g of $\mathscr{G}$, therefore, there corresponds a linear mapping $\Psi(g)$ of $\mathscr{W}$ onto itself, i.e., $\Psi(g) X = X_1$, that is valid for every state b,

$$\Psi(g)\, X \cdot b = X \cdot \Theta(g^{-1})\, b\,.$$

Thus if $\mathscr{W}$ is a finite-dimensional vector space, then a function Ψ determines a matrix representation of $\mathscr{W}$.

It can be shown (GILES, 1964, pp. 134—135) that an in the case of Galilean thermodynamics an invariant entropy function may not exist. An equilibrium state involves a uniform homogeneous expansion or contraction in this case. However, the entropy function is in fact invariant under every proper Lorenz transformation (GILES, 1964, pp. 175—177).

Now, it is shown by GILES (1964, pp. 160—161), that if X is a *component of content* (conserved quantity) then $\Psi(g) X$ is one also, for any element g of $\mathscr{G}$. Thus the subspace $\mathscr{V}$ of $\mathscr{W}$ is invariant under $\Psi(g)$. Similarly, if X is an *entropy function* then so is $\Psi(g) X$ and the subspace $\mathscr{X}$ of $\mathscr{W}$ is invariant under $\Psi(g)$.

Suppose $\mathscr{V}$ is n-dimensional and $\mathscr{W}$ has a base $(S, Q_1, \ldots, Q_n)$ where S is an entropy function and $Q_1, \ldots, Q_n$ are components of content, then any element of $\mathscr{W}$ is then represented by a column matrix $(a_0, a_1, \ldots, a_n)$ and belongs to $\mathscr{V}$ if $a_0 = 0$ or to $\mathscr{X}$ if $a_1, \ldots, a_n = 0$. For each element g of $\mathscr{G}$ the linear operator $\Psi(g)$ may then be written as a square matrix of the form shown in Fig. 4. (GILES, 1964, p. 161): where $\nu(g)$ is a square matrix of order n and $\lambda(g)$ is a real numerical function of g. As Ψ is an $(n+1)$-dimensional representation of $\mathscr{G}$, it follows that ν is an n-dimensional representation of $\mathscr{G}$, and λ is a 1-dimensional representation of $\mathscr{G}$.

The point to be emphasized is that this analysis of a matrix representation of the symmetry group $\mathscr{G}$ is accomplished *without regard* to thermodynamics. If, however, $\mathscr{G} = \mathscr{G}_L$ where $\mathscr{G}_L$ is the inhomogeneous Lorenz group and ν of $\mathscr{V}$ is an adjoint representation, it can be shown (GILES, 1964, pp. 175—177) that the representation Ψ in the vector space $\mathscr{W}$ is necessarily the direct sum of ν, the representation in the vector space $\mathscr{V}$, and the trivial 1-dimensional representation. This means: there always exists an entropy function which is invariant under every Lorenz transformation.

If this is the case then there always exists a *negative entropy* function which is invariant under an analogous Lorenz transformation. This is because the symmetry group was derived without respect to thermodynamics.

If a Lorenz transformation is indeed needed in the realm of information theory in order that information structures be adequately studied, then remembering that we are dealing with space-time, it would seem that it is sufficient to study $\boldsymbol{E}$ the electric field. This is because the resolution of the electromagnetic field into electric and magnetic components is wholly dependent on the motion of the observer. Thus an observer moving with a fixed charge observes a purely electrostatic field but an observer, relative to whom the charge is moving, sees a magnetic field and identifies the moving charge with current (JONES, 1964, p. 129). By the study of the inhomogeneous Lorenz group, $\mathscr{G}_L$, or rather its analogy in information theory,

$$\psi(g) = \left[\begin{array}{c|c} \lambda(g) & 0 \\ \hline \mu(g) & \nu(g) \end{array}\right]$$

Fig. 4. By permission

we may be concerned solely with $\boldsymbol{E}$, space (in x, y and z coordination), and time. This is not beyond the capabilities of present-day electrophysiology.

Conclusion

1. Cognitive structures have a symmetrical aspect.

2. In the engenderment of cognitive structures, there exists a metron-logon content exchange developing into the informational equivalent of a Carnot cycle.

3. Lorenz transformations are needed to describe the changes which leave negative entropy invariant.

4. Equations analogous to EINSTEIN's field equations must be devised in order that the relations between structural regulation of biological processes and energic be defined.

5. Lorenz transformations within the confines of an abstract symmetry group may be described by electrical potential, spatial coordinates and time. This kind of study is not beyond the capabilities of present-day electrophysiology.

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